



**University of
Zurich**^{UZH}

**Zurich Open Repository and
Archive**

University of Zurich
University Library
Strickhofstrasse 39
CH-8057 Zurich
www.zora.uzh.ch

Year: 2009

Two-loop QCD helicity amplitudes for (2+1)-jet production in deep inelastic scattering

Gehrmann, T ; Glover, E W N

Abstract: We derive the two-loop QCD helicity amplitudes for the processes $q \rightarrow qg$ (View the MathML source) and View the MathML source, which are the partonic reactions yielding (2+1)-jet final states in deep inelastic lepton nucleon scattering. The amplitudes are obtained by analytic continuation of the known helicity amplitudes for View the MathML source. We separate the infrared divergent and finite parts of the amplitudes using Catani's infrared factorization formula. The analytic results for the finite parts of the amplitudes are expressed in terms of one- and two-dimensional harmonic polylogarithms. To evaluate these functions numerically, we list in detail the non-trivial (and kinematic region dependent) variable transformations one needs to perform.

DOI: <https://doi.org/10.1016/j.physletb.2009.04.083>

Posted at the Zurich Open Repository and Archive, University of Zurich

ZORA URL: <https://doi.org/10.5167/uzh-30921>

Journal Article

Accepted Version

Originally published at:

Gehrmann, T; Glover, E W N (2009). Two-loop QCD helicity amplitudes for (2+1)-jet production in deep inelastic scattering. *Physics Letters B*, 676(4-5):146-151.

DOI: <https://doi.org/10.1016/j.physletb.2009.04.083>

Two-Loop QCD Helicity Amplitudes for (2+1)-Jet Production in Deep Inelastic Scattering

T. Gehrmann^a and E.W.N. Glover^b

^a *Institut für Theoretische Physik, Universität Zürich, Winterthurerstrasse 190,
CH-8057 Zürich, Switzerland*

^b *Institute for Particle Physics Phenomenology, University of Durham, South Road,
Durham DH1 3LE, England*

Abstract

We derive the two-loop QCD helicity amplitudes for the processes $\ell q \rightarrow \ell q g$ ($\ell \bar{q} \rightarrow \ell \bar{q} g$) and $\ell g \rightarrow \ell q \bar{q}$, which are the partonic reactions yielding $(2+1)$ -jet final states in deep inelastic lepton nucleon scattering. The amplitudes are obtained by analytic continuation of the known helicity amplitudes for $e^+e^- \rightarrow q\bar{q}g$. We separate the infrared divergent and finite parts of the amplitudes using Catani's infrared factorization formula. The analytic results for the finite parts of the amplitudes are expressed in terms of one- and two-dimensional harmonic polylogarithms. To evaluate these functions numerically, we list in detail the non-trivial (and kinematic region dependent) variable transformations one needs to perform.

1 Introduction

The study of the hadronic final state in deep inelastic lepton-nucleon scattering (DIS) allows the determination of aspects of the nucleon structure which are not accessible in inclusive scattering as well as testing predictions of quantum chromodynamics (QCD) to a high accuracy. In particular, the two-plus-one-jet production [1] rate in DIS can be used both to probe the gluon distribution and to enable a precise determination of the strong coupling constant α_s .

The measurement [4, 5] of these jet observables at the electron-proton collider HERA, with a much larger kinematical range than at earlier fixed target experiments, allows precision studies, and the inclusion of next-to-leading order (NLO) corrections [2, 3] has become mandatory. However, the present data already highlight the limitations of the NLO description: the error on the extraction of α_s from HERA (2+1) jet data is dominated [6] not by the statistical uncertainty on the data, but by the uncertainty inherent to the NLO calculation, as estimated by varying renormalization and factorization scales. Given that the statistical precision of the data will further improve once all data from the HERA-II run are analysed, the theoretical description requires the inclusion of the next-to-next-to-leading order (NNLO), i.e. $\mathcal{O}(\alpha_s^3)$, corrections.

The dijet production cross section in deep inelastic scattering at HERA is very sensitive to the gluon distribution in the proton. However, present determinations of parton distribution functions [7] at NNLO accuracy [8] can only include data sets for observables where NNLO corrections are known [9]. Consequently, the precise HERA deep inelastic dijet data could be used in global NNLO determinations only once the NNLO QCD corrections to this observable are computed.

In terms of matrix elements, the calculation of these corrections requires the computation of three contributions: the tree level $\gamma^*i \rightarrow 4$ partons amplitudes, the one-loop corrections to the $\gamma^*i \rightarrow 3$ partons amplitudes and the two-loop corrections to the $\gamma^*i \rightarrow 2$ partons amplitudes, where $i = q, g$. All of these amplitudes can be obtained by analytic continuation from the respective amplitudes contributing to $e^+e^- \rightarrow 3$ jets at NNLO: the tree level $\gamma^* \rightarrow 5$ partons [10], the one-loop corrections to the $\gamma^* \rightarrow 4$ partons amplitudes [11], and the two-loop corrections to the $\gamma^* \rightarrow 3$ partons amplitudes [12, 13]. The analytic continuation of the tree level and one-loop amplitudes is rather straightforward, it has already been carried out in the context of the NLO corrections to the $(3+1)$ -jet rate in DIS [14]. The analytic continuation of the two-loop amplitudes is more involved and is the topic of the present paper.

Corrections to the deep inelastic $(2+1)$ -jet cross section, which are the phenomenologically most relevant applications, require only the computation of the helicity averaged squared matrix element. However, the helicity amplitudes derived here can be applied to compute angular correlations between the outgoing lepton direction and the orientation of the $(2+1)$ -jet system [15]. They also allow the calculation of jet cross sections in polarized lepton-nucleon collisions [16] which can constrain the polarized parton distributions in the nucleon.

Even though the matrix elements for all NNLO partonic subprocesses contributing to $(2+1)$ -jet final states in DIS are now available, additional work is still needed before quantitative predictions can be made. The two-loop amplitudes computed here must be combined with the one-loop corrections to $\gamma^*i \rightarrow 3$ partons, where one of the partons becomes collinear or soft, as well as tree-level processes $\gamma^*i \rightarrow 4$ partons with two soft or collinear partons in a way that explicitly allows all of the infrared singularities to cancel one another. This task is usually accomplished by a subtraction method, which exploits the universal factorization properties of QCD amplitudes in infrared soft or collinear limits to construct real radiation subtraction terms. These subtraction terms approximate the full real radiation matrix elements in all unresolved limits, but are sufficiently simple to be integrated analytically. Consequently, they can be combined with the virtual loop corrections to the matrix elements to yield infrared finite results for suitably defined observables. Several general subtraction schemes have been proposed at NLO [17].

For observables with only final state partons, an NNLO subtraction formalism, antenna subtraction, has been established in [18]. The antenna subtraction formalism constructs the subtraction terms from antenna functions, which are derived systematically from physical matrix elements [19]. This formalism has been applied in the computation of NNLO corrections to three-jet production in electron-positron annihilation [20–22] and related event shapes [23, 24]. This formalism can be extended to include parton showers at higher orders [25], thereby offering a process independent matching of fixed-order calculations and logarithmic resummations [26, 27], which had to be done on a case-by-case basis for individual observables [28] up to now.

For processes with initial-state partons, antenna subtraction has been fully worked out only to NLO so far [29], work towards an extension to NNLO is in progress.

Other approaches to perform NNLO calculations of exclusive observables with initial state partons are the use of sector decomposition and a subtraction method based on the transverse momentum structure of the final state. The sector decomposition algorithm [30] analytically decomposes both phase space and loop integrals into their Laurent expansion in dimensional regularization, and performs a subsequent numerical computation of the coefficients of this expansion. Using this formalism, NNLO results were obtained for Higgs production [31] and vector boson production [32] at hadron colliders. Both reactions were equally computed independently using an NNLO subtraction formalism exploiting the specific transverse momentum structure of these observables [33].

This paper is structured as follows: in Section 2, we describe the kinematical situation relevant to DIS-(2 + 1)-jet production. The two-loop helicity amplitudes are computed by analytic continuation from the $\gamma^* \rightarrow 3$ partons amplitudes in Section 3. Finally, Section 4 contains a discussion of the results and conclusions.

2 Kinematics

The partonic subprocesses contributing to the production of two hard jets (besides the current jet) in DIS are lepton-quark scattering

$$\ell(p_5) + q(-p_2) \longrightarrow \ell(-p_6) + q(p_1) + g(p_3) , \quad (2.1)$$

(with lepton-antiquark scattering following trivially from charge conjugation) and lepton-gluon scattering

$$\ell(p_5) + g(-p_3) \longrightarrow \ell(-p_6) + q(p_1) + \bar{q}(p_2) . \quad (2.2)$$

The momentum of the vector boson mediating the interaction is given by

$$-q^\mu = -p_6^\mu - p_5^\mu . \quad (2.3)$$

It is convenient to define the invariants

$$s_{12} = (p_1 + p_2)^2 , \quad s_{13} = (p_1 + p_3)^2 , \quad s_{23} = (p_2 + p_3)^2 , \quad (2.4)$$

which fulfil

$$q^2 = (p_1 + p_2 + p_3)^2 = s_{12} + s_{13} + s_{23} \equiv s_{123} , \quad (2.5)$$

as well as the dimensionless invariants

$$x = s_{12}/s_{123} , \quad y = s_{13}/s_{123} , \quad z = s_{23}/s_{123} , \quad (2.6)$$

which satisfy $x + y + z = 1$.

In $e^+e^- \rightarrow 3$ jet ($3j$) production, q^2 is time-like (hence positive) and all the s_{ij} are also positive, which implies that x, y, z all lie in the interval $[0; 1]$, with the above constraint $x + y + z = 1$. For DIS-(2 + 1) j production, q^2 is space-like (hence negative) and for lepton-quark scattering (2.1) the invariants fulfil

$$q^2 < 0 , \quad s_{23} > 0 , \quad s_{12} < 0 , \quad s_{13} < 0 , \quad (2.7)$$

or, equivalently,

$$x > 0 , \quad y > 0 , \quad z < 0 . \quad (2.8)$$

For lepton-gluon scattering (2.2), one finds

$$q^2 < 0 , \quad s_{12} > 0 , \quad s_{13} < 0 , \quad s_{23} < 0 , \quad (2.9)$$

respectively,

$$x < 0 , \quad y > 0 , \quad z > 0 . \quad (2.10)$$

It is worth noting that the kinematical regions for these two processes are further subdivided by cuts in $x = 1$ and $y = 1$ (lepton-quark scattering) or $y = 1$ and $z = 1$ (lepton-gluon scattering), as was first pointed

out by Graudenz in the context of the calculation of the NLO matrix elements [2]. As a consequence, it is not possible to find an analytic expression for the amplitudes, which covers the whole kinematic region *and* does not contain functions with implicit imaginary parts. We shall see below that the most convenient representation is obtained by subdividing the kinematical plane for each process according to the cuts into four sectors. In each of these sectors, we obtain an analytic expression for the amplitudes with all imaginary parts made explicit; joining the different regions together, one obtains a result which is continuous, but not analytic, across the cuts.

3 Helicity amplitudes

The renormalized amplitude $|\mathcal{M}\rangle$ can be written as

$$|\mathcal{M}\rangle = V^\mu \mathcal{S}_\mu(q; g; \bar{q}) , \quad (3.1)$$

where V^μ represents the lepton current and \mathcal{S}_μ denotes the hadron current. With \bar{q} , we denote the incoming quark (momentum $-p_2$) in lepton-quark scattering or the outgoing antiquark (momentum $+p_2$) in lepton-gluon scattering. A convenient method to evaluate the helicity amplitudes is in terms of Weyl-van der Waerden spinors, which is described briefly in the appendix of [13] and in detail in [34, 35].

Using the Weyl-van der Waerden spinor calculus, we can express the lepton current V_μ for photon exchange in terms of the helicities of the incoming $l^-(p_5)$ and outgoing $l^-(p_6)$. Explicitly,

$$V_\mu^\gamma(l^-(p_5)-, l^-(p_6)-) = e\sigma_\mu^{\dot{A}B} p_{6\dot{A}} p_{5B} \frac{L_{ll}^\gamma}{p_4^2} , \quad V_\mu^\gamma(l^-(p_5)+, l^-(p_6)+) = e\sigma_\mu^{\dot{A}B} p_{5\dot{A}} p_{6B} \frac{R_{ll}^\gamma}{p_4^2} , \quad (3.2)$$

where for photon exchange,

$$L_{f_1 f_2}^\gamma = R_{f_1 f_2}^\gamma = -e_{f_1} \delta_{f_1 f_2} . \quad (3.3)$$

It is straightforward to account for charged and neutral current exchange by appropriate replacement of the couplings, keeping in mind that in these cases left- and right-handed couplings are no longer identical. The lepton current for incident anti-leptons follows from charge conjugation.

The hadronic current \mathcal{S}_μ is related to the fixed helicity currents, $\mathcal{S}_{\dot{A}B}$, by

$$\mathcal{S}_\mu(q+; g\lambda; \bar{q}-) = R_{f_1 f_2}^\gamma \sqrt{2} \sigma_\mu^{\dot{A}B} \mathcal{S}_{\dot{A}B}(q+; g\lambda; \bar{q}-) , \quad (3.4)$$

$$\mathcal{S}_\mu(q-; g\lambda; \bar{q}+) = L_{f_1 f_2}^\gamma \sqrt{2} \sigma_\mu^{\dot{A}B} \mathcal{S}_{\dot{A}B}(q-; g\lambda; \bar{q}+) . \quad (3.5)$$

As above, the gauge boson coupling is extracted from $\mathcal{S}_{\dot{A}B}$, and charged and neutral current reactions follow from appropriate substitutions.

The currents with the quark helicities flipped follow from parity conservation:

$$\mathcal{S}_{\dot{A}B}(q-; g\lambda; \bar{q}+) = (\mathcal{S}_{\dot{B}A}(q+; g(-\lambda); \bar{q}-))^* . \quad (3.6)$$

Charge conjugation implies the following relations between currents with different helicities:

$$\mathcal{S}_{\dot{A}B}(q\lambda_q; g\lambda; \bar{q}\lambda_{\bar{q}}) = (-1) \mathcal{S}_{\dot{A}B}(\bar{q}\lambda_{\bar{q}}; g\lambda; q\lambda_q) . \quad (3.7)$$

All helicity amplitudes are therefore related to the amplitudes with $\lambda_q = +$ and $\lambda_{\bar{q}} = -$. Explicitly, we find

$$\begin{aligned} \mathcal{S}_{\dot{A}B}(q+; g+; \bar{q}-) &= \alpha(x, y, z) \frac{p_{1\dot{A}D} p_2^D p_{2B}}{\langle p_1 p_3 \rangle \langle p_3 p_2 \rangle} + \beta(x, y, z) \frac{p_{3\dot{A}D} p_2^D p_{2B}}{\langle p_1 p_3 \rangle \langle p_3 p_2 \rangle} + \gamma(x, y, z) \frac{p_{1\dot{C}B} p_3^{\dot{C}} p_{3\dot{A}}}{\langle p_1 p_3 \rangle \langle p_3 p_2 \rangle^*} \\ &+ \delta(x, y, z) \frac{\langle p_1 p_3 \rangle^*}{\langle p_1 p_3 \rangle \langle p_1 p_2 \rangle^*} (p_{1\dot{A}B} + p_{2\dot{A}B} + p_{3\dot{A}B}) . \end{aligned} \quad (3.8)$$

The other helicity amplitudes are obtained from $\mathcal{S}_{\dot{A}B}(q+; g+; \bar{q}-)$ by the above parity and charge conjugation relations. Current conservation implies the following relation between the four helicity coefficients,

$$\alpha(x, y, z) - \beta(x, y, z) - \gamma(x, y, z) - \frac{2s_{123}}{s_{12}} \delta(x, y, z) = 0 . \quad (3.9)$$

Equation (3.9) suffices to fix $\delta(x, y, z)$.

The three remaining helicity amplitude coefficients α , β and γ are vectors in colour space and have perturbative expansions:

$$\Omega = \sqrt{4\pi\alpha}\sqrt{4\pi\alpha_s} \mathbf{T}_{ij}^a \left[\Omega^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \Omega^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Omega^{(2)} + \mathcal{O}(\alpha_s^3) \right], \quad (3.10)$$

for $\Omega = \alpha, \beta, \gamma$ where the dependence on (x, y, z) is implicit.

The helicity coefficients were extracted from a Feynman diagram calculation in dimensional regularization [36, 37], with $d = 4 - 2\epsilon$ space-time dimensions, by applying suitable projectors to the amplitudes at the required order in perturbation theory. Once these projectors are applied to expose the helicity structure, one is left with the task of computing a large number of two-loop integrals. Using integration-by-parts [37, 38] and Lorentz-invariance [39] identities, these were reduced [40] (making extensive use of MAPLE [41] and FORM [42]) to a small number of so-called master integrals, which were derived in [43]. Insertion of these master integrals into the contraction of Feynman amplitudes with the projectors then yields the unrenormalized helicity amplitudes [13]. A partial check was made in [44], where two of the seven colour factors contributing to the $\gamma^* \rightarrow 3$ partons two-loop helicity amplitudes were derived by direct evaluation of the integrals (i.e. not using a reduction to master integrals), yielding full agreement with [13].

The $3j$ amplitudes obtained in [13] are expressed in terms of two-dimensional harmonic polylogarithms (2dHPLs). The 2dHPLs are an extension of the harmonic polylogarithms (HPLs) of [45]. Numerical routines providing an evaluation of the HPLs [46, 47] and 2dHPLs [48] are available. The implementations apply to HPLs of arbitrary real [45, 46] or complex [47] arguments, but only for a limited range of arguments of the 2dHPLs. This range of arguments corresponds precisely to the $3j$ kinematics. Formulae for the analytic continuation of HPLs and 2dHPLs to ranges of argument relevant to all $2 \rightarrow 2$ scattering kinematics were derived in [49]. Given the restricted range of arguments for the numerical implementation of the 2dHPLs [48], in [49] transformations to map all $2 \rightarrow 2$ scattering kinematics into this range of arguments were derived. These analytical continuations were subsequently confirmed by direct numerical evaluation [50] of the relevant master integrals [43] in the relevant kinematical regions using the numerical Mellin-Barnes technique. Using the results of [49], we can perform the analytic continuation of all the unrenormalized helicity amplitudes from the $\gamma^* \rightarrow 3$ partons kinematics considered in [12] to the kinematical situation of DIS-(2 + 1)-jet production.

Subsequently, renormalization is performed in the $\overline{\text{MS}}$ -scheme by replacing the bare coupling α_0 with the renormalized coupling $\alpha_s \equiv \alpha_s(\mu^2)$, evaluated at the renormalization scale μ^2

$$\alpha_0 \mu_0^{2\epsilon} S_\epsilon = \alpha_s \mu^{2\epsilon} \left[1 - \frac{\beta_0}{\epsilon} \left(\frac{\alpha_s}{2\pi}\right) + \left(\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{2\epsilon}\right) \left(\frac{\alpha_s}{2\pi}\right)^2 + \mathcal{O}(\alpha_s^3) \right], \quad (3.11)$$

where

$$S_\epsilon = (4\pi)^\epsilon e^{-\epsilon\gamma} \quad \text{with Euler constant } \gamma = 0.5772\dots$$

and μ_0^2 is the mass parameter introduced in dimensional regularization [36, 37] to maintain a dimensionless coupling in the bare QCD Lagrangian density; β_0 and β_1 are the first two coefficients of the QCD β -function:

$$\beta_0 = \frac{11C_A - 4T_R N_F}{6}, \quad \beta_1 = \frac{17C_A^2 - 10C_A T_R N_F - 6C_F T_R N_F}{6}, \quad (3.12)$$

with the QCD colour factors

$$C_A = N, \quad C_F = \frac{N^2 - 1}{2N}, \quad T_R = \frac{1}{2}. \quad (3.13)$$

For the remainder of this paper we will set the renormalization scale $\mu^2 = -q^2$.

This procedure yields the renormalized tensor coefficients,

$$\begin{aligned} \Omega^{(0)} &= \Omega^{(0),\text{un}}, \\ \Omega^{(1)} &= S_\epsilon^{-1} \Omega^{(1),\text{un}} - \frac{\beta_0}{2\epsilon} \Omega^{(0),\text{un}}, \\ \Omega^{(2)} &= S_\epsilon^{-2} \Omega^{(2),\text{un}} - \frac{3\beta_0}{2\epsilon} S_\epsilon^{-1} \Omega^{(1),\text{un}} - \left(\frac{\beta_1}{4\epsilon} - \frac{3\beta_0^2}{8\epsilon^2}\right) \Omega^{(0),\text{un}}. \end{aligned} \quad (3.14)$$

The full scale dependence of the renormalized helicity coefficients can be recovered from the renormalization group equation:

$$\begin{aligned}\Omega &= \sqrt{4\pi\alpha}\sqrt{4\pi\alpha_s} \mathbf{T}_{ij}^a \left\{ \Omega^{(0)} + \left(\frac{\alpha_s(\mu^2)}{2\pi} \right) \left[\Omega^{(1)} + \frac{\beta_0}{2} \Omega^{(0)} \ln \left(\frac{\mu^2}{q^2} \right) \right] \right. \\ &\quad \left. + \left(\frac{\alpha_s(\mu^2)}{2\pi} \right)^2 \left[\Omega^{(2)} + \left(\frac{3\beta_0}{2} \Omega^{(1)} + \frac{\beta_1}{2} \Omega^{(0)} \right) \ln \left(\frac{\mu^2}{q^2} \right) + \frac{3\beta_0^2}{8} \Omega^{(0)} \ln^2 \left(\frac{\mu^2}{q^2} \right) \right] + \mathcal{O}(\alpha_s^3) \right\}. \quad (3.15)\end{aligned}$$

After performing ultraviolet renormalization, the amplitudes still contain singularities, which are of infrared origin and will be analytically cancelled by those occurring in radiative processes of the same order. Catani [51] has shown how to organize the infrared pole structure of the one- and two-loop contributions renormalized in the $\overline{\text{MS}}$ scheme in terms of the tree and renormalized one-loop amplitudes. The same procedure applies to the tensor coefficients:

$$\begin{aligned}\Omega^{(1)} &= \mathbf{I}^{(1)}(\epsilon) \Omega^{(0)} + \Omega^{(1),\text{finite}}, \\ \Omega^{(2)} &= \left(-\frac{1}{2} \mathbf{I}^{(1)}(\epsilon) \mathbf{I}^{(1)}(\epsilon) - \frac{\beta_0}{\epsilon} \mathbf{I}^{(1)}(\epsilon) + e^{-\epsilon\gamma} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\beta_0}{\epsilon} + K \right) \mathbf{I}^{(1)}(2\epsilon) + \mathbf{H}^{(2)}(\epsilon) \right) \Omega^{(0)} \\ &\quad + \mathbf{I}^{(1)}(\epsilon) \Omega^{(1)} + \Omega^{(2),\text{finite}}, \quad (3.16)\end{aligned}$$

where the constant K is

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} T_R N_F. \quad (3.17)$$

For this particular process, there is only one colour structure present at tree level which, in terms of the gluon colour a and the quark and antiquark colours i and j , is simply \mathbf{T}_{ij}^a . Adding higher loops does not introduce additional colour structures, and the amplitudes are therefore vectors in a one-dimensional space. Similarly, the infrared singularity operator $\mathbf{I}^{(1)}(\epsilon)$ is a scalar in colour space and is given by

$$\mathbf{I}^{(1)}(\epsilon) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \left[N \left(\frac{1}{\epsilon^2} + \frac{3}{4\epsilon} + \frac{\beta_0}{2N\epsilon} \right) (\mathbf{S}_{13} + \mathbf{S}_{23}) - \frac{1}{N} \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right) \mathbf{S}_{12} \right], \quad (3.18)$$

where (since we have set $\mu^2 = -s_{123}$)

$$\mathbf{S}_{ij} = \left(\frac{s_{123}}{s_{ij}} \right)^\epsilon. \quad (3.19)$$

Using the invariants s_{ij} as defined in Section 2, phase factors distinguishing initial and final state partons become obsolete in this expression. Note that on expanding \mathbf{S}_{ij} , imaginary parts are generated, the sign of which is fixed by the small imaginary part $+i0$ of all s_{ij} .

Finally, the term of Eq. (3.16) that involves $\mathbf{H}^{(2)}(\epsilon)$ produces only a single pole in ϵ and is given by

$$\mathbf{H}^{(2)}(\epsilon) = \frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)} \mathbf{H}^{(2)}, \quad (3.20)$$

where the constant $\mathbf{H}^{(2)}$ is renormalization-scheme-dependent. As with the single-pole parts of $\mathbf{I}^{(1)}(\epsilon)$, the process-dependent $\mathbf{H}^{(2)}$ can be constructed by counting the number of radiating partons present in the event. In our case, in the $\overline{\text{MS}}$ scheme:

$$\begin{aligned}H^{(2)} &= \left(4\zeta_3 + \frac{589}{432} - \frac{11\pi^2}{72} \right) N^2 + \left(-\frac{1}{2}\zeta_3 - \frac{41}{54} - \frac{\pi^2}{48} \right) + \left(-3\zeta_3 - \frac{3}{16} + \frac{\pi^2}{4} \right) \frac{1}{N^2} \\ &\quad + \left(-\frac{19}{18} + \frac{\pi^2}{36} \right) N N_F + \left(-\frac{1}{54} - \frac{\pi^2}{24} \right) \frac{N_F}{N} + \frac{5}{27} N_F^2. \quad (3.21)\end{aligned}$$

At leading order

$$\alpha^{(0)}(x, y, z) = \beta^{(0)}(x, y, z) = 1 \quad \text{and} \quad \gamma^{(0)}(x, y, z) = 0. \quad (3.22)$$

The renormalized next-to-leading order helicity amplitude coefficients can be straightforwardly obtained to all orders in ϵ from the known exact expressions for the one-loop master integrals. For practical purposes, they are needed through to $\mathcal{O}(\epsilon^2)$ in evaluating the infrared-divergent one-loop contribution to the two-loop amplitude, while only the finite piece is needed for the one-loop self-interference. They can be decomposed according to their colour structure as follows:

$$\Omega^{(1),\text{finite}}(x, y, z) = N a_\Omega(x, y, z) + \frac{1}{N} b_\Omega(x, y, z) + \beta_0 c_\Omega(x, y, z) . \quad (3.23)$$

The expansion of the coefficients through to ϵ^2 yields HPLs and 2dHPLs up to weight 4 for a_Ω , b_Ω and up to weight 3 for c_Ω . In order to obtain expressions suitable for numerical evaluation (which requires the arguments of the 2dHPLs to be in a restricted range) and to make all imaginary parts explicit, we use the following decomposition [2] for the coefficients $l = a, b, c$ of the individual colour factors:

$$\begin{aligned} l_\Omega(x, y, z) = & \Theta(x)\Theta(1-x)\Theta(y)\Theta(1-y)\Theta(-z)l_{\Omega,1}(x+z, -z) \\ & + \Theta(x-1)\Theta(y)\Theta(1-y)\Theta(-z)l_{\Omega,2}\left(-\frac{y+z}{x}, \frac{y}{x}\right) \\ & + \Theta(x)\Theta(1-x)\Theta(y-1)\Theta(-z)l_{\Omega,3}\left(\frac{y+z}{y}, \frac{x}{y}\right) \\ & + \Theta(x-1)\Theta(y-1)\Theta(-z)l_{\Omega,4}\left(\frac{y+z}{z}, -\frac{1}{z}\right) \end{aligned} \quad (3.24)$$

for the lepton–quark scattering process (2.1) and

$$\begin{aligned} l_\Omega(x, y, z) = & \Theta(z)\Theta(1-z)\Theta(y)\Theta(1-y)\Theta(-x)l_{\Omega,5}(x+z, -x) \\ & + \Theta(z-1)\Theta(y)\Theta(1-y)\Theta(-x)l_{\Omega,6}\left(-\frac{x+y}{z}, \frac{y}{z}\right) \\ & + \Theta(z)\Theta(1-z)\Theta(y-1)\Theta(-x)l_{\Omega,7}\left(-\frac{x+z}{y}, \frac{z}{y}\right) \\ & + \Theta(z-1)\Theta(y-1)\Theta(-x)l_{\Omega,8}\left(\frac{x+y}{x}, -\frac{1}{x}\right) \end{aligned} \quad (3.25)$$

for the lepton–gluon scattering process (2.2).

The coefficients $l_{\Omega,i}(r, s)$ are then expressed in terms of HPLs of argument s and 2dHPLs of argument r , with s featuring in the index vector. The above choices of argument ensure that $0 \leq r \leq 1-s$ and $0 \leq s \leq 1$, as required for the numerical evaluation of the 2dHPLs [48]. The explicit expressions are of considerable size, such that we shall not quote them here. An example of the size and structure of those coefficients can be found in [12], where we explicitly list the helicity-averaged one-loop times one-loop and tree times two-loop matrix elements for $e^+e^- \rightarrow 3$ jets. It should be noted that these finite pieces of the one-loop coefficients can equally well be written in terms of ordinary logarithms and dilogarithms, see [52, 53]. The reason for expressing them in terms of HPLs and 2dHPLs here is their usage in the infrared counter-term of the two-loop coefficients, which cannot be fully expressed in terms of logarithmic and polylogarithmic functions.

The finite two-loop remainder is obtained by subtracting the predicted infrared structure (expanded through to $\mathcal{O}(\epsilon^0)$) from the renormalized helicity coefficient. We further decompose the finite remainder according to the colour structure, as follows:

$$\begin{aligned} \Omega^{(2),\text{finite}}(x, y, z) = & N^2 A_\Omega(x, y, z) + B_\Omega(x, y, z) + \frac{1}{N^2} C_\Omega(x, y, z) + N N_F D_\Omega(x, y, z) \\ & + \frac{N_F}{N} E_\Omega(x, y, z) + N_F^2 F_\Omega(x, y, z) + N_{F,V} \left(\frac{4}{N} - N \right) G_\Omega(x, y, z) , \end{aligned} \quad (3.26)$$

where the last term is generated by graphs where the virtual gauge boson does not couple directly to the final-state quarks. This contribution is denoted by $N_{F,V}$ and is proportional to the charge weighted sum of

the quark flavours. In the case of purely electromagnetic interactions we find,

$$N_{F,\gamma} = \frac{\sum_q e_q}{e_q}. \quad (3.27)$$

Including electroweak vector boson interactions, the same class of diagrams yields not only a contribution from the vector component (obtained by appropriate replacement of the couplings in the above formula [13]), but also a contribution involving the axial couplings of the vector boson under consideration. This contribution vanishes however if summed over isospin doublets.

We apply the same decomposition as used at one loop for the coefficients $L = A, \dots, G$ of the individual colour factors:

$$\begin{aligned} L_\Omega(x, y, z) = & \Theta(x)\Theta(1-x)\Theta(y)\Theta(1-y)\Theta(-z)L_{\Omega,1}(x+z, -z) \\ & + \Theta(x-1)\Theta(y)\Theta(1-y)\Theta(-z)L_{\Omega,2}\left(-\frac{y+z}{x}, \frac{y}{x}\right) \\ & + \Theta(x)\Theta(1-x)\Theta(y-1)\Theta(-z)L_{\Omega,3}\left(\frac{y+z}{y}, \frac{x}{y}\right) \\ & + \Theta(x-1)\Theta(y-1)\Theta(-z)L_{\Omega,4}\left(\frac{y+z}{z}, -\frac{1}{z}\right) \end{aligned} \quad (3.28)$$

for the lepton-quark scattering process (2.1) and

$$\begin{aligned} L_\Omega(x, y, z) = & \Theta(z)\Theta(1-z)\Theta(y)\Theta(1-y)\Theta(-x)L_{\Omega,5}(x+z, -x) \\ & + \Theta(z-1)\Theta(y)\Theta(1-y)\Theta(-x)L_{\Omega,6}\left(-\frac{x+y}{z}, \frac{y}{z}\right) \\ & + \Theta(z)\Theta(1-z)\Theta(y-1)\Theta(-x)L_{\Omega,7}\left(-\frac{x+z}{y}, \frac{z}{y}\right) \\ & + \Theta(z-1)\Theta(y-1)\Theta(-x)L_{\Omega,8}\left(\frac{x+y}{x}, -\frac{1}{x}\right) \end{aligned} \quad (3.29)$$

for the lepton-gluon scattering process (2.2).

The helicity coefficients contain HPLs and 2dHPLs up to weight 4 in the A, B, C, G -terms, up to weight 3 in the D, E -terms and up to weight 2 in the F -term. The size of each helicity coefficient is comparable to the size of the helicity-averaged tree times two-loop matrix element quoted in [12]. We therefore refrain from quoting them explicitly. The complete set of one-loop and two-loop coefficients in FORM and FORTRAN format is provided with the sources of the paper on the archive <http://arxiv.org>.

4 Conclusions and Outlook

In this paper, we have applied the analytic continuation procedures for two-loop four-point functions with one off-shell leg derived in [49] to obtain the two-loop helicity amplitudes relevant to DIS-(2+1) jet production from the amplitudes for $\gamma^* \rightarrow q\bar{q}g$ derived earlier. Besides enabling the computation of the virtual two-loop corrections to the DIS-(2+1) jet cross section and related event shape observables (which follow by interfering the amplitudes with their tree level counterparts and summing over all external helicities), knowledge of the helicity amplitudes provides additional information about the scattering process. In particular, oriented event shapes as well as jet rates in polarized DIS can be computed from them. These matrix elements represent one of the missing ingredients for NNLO computations of deep inelastic jet production. Such calculations require the construction of a parton-level event generator program, which contains all partonic channels contributing to the observable under consideration. Since the one-loop and tree-level contributions at NNLO contain more partons than hard jets, they will contribute infrared real radiation singularities, which have to be subtracted in the corresponding partonic channels, and added to the two-loop virtual contribution to yield finite expressions suitable for numerical evaluation. While a formalism for NNLO subtraction is well established for processes involving only final-state partons, some extra work is required to subtract singularities arising from unresolved emission off initial-state partons. Work in this direction is ongoing.

Acknowledgements

We would like to thank Lee Garland, Thanos Koukoutsakis and particularly Ettore Remiddi for their collaboration on earlier work that lead towards this paper. EWNG gratefully acknowledges the support of the Wolfson Foundation and the Royal Society. This research was supported in part by the Swiss National Science Foundation (SNF) under contract 200020-117602, by the UK Science and Technology Facilities Council and by the European Commission’s Marie-Curie Research Training Network under contract MRTN-CT-2006-035505 “Tools and Precision Calculations for Physics Discoveries at Colliders”.

References

- [1] P. M. Stevenson, Nucl. Phys. B **150**, 357 (1979);
K. H. Streng, T. F. Walsh and P. M. Zerwas, Z. Phys. C **2** (1979) 237;
P. Binetruy and G. Girardi, Nucl. Phys. B **155** (1979) 150;
C. Rumpf, G. Kramer and J. Willrodt, Z. Phys. C **7** (1981) 337.
- [2] D. Graudenz, Phys. Lett. B **256** (1991) 518; Phys. Rev. D **49** (1994) 3291 [hep-ph/9307311].
- [3] E. Mirkes and D. Zeppenfeld, Phys. Lett. B **380** (1996) 205 [hep-ph/9511448];
S. Catani and M. H. Seymour, Phys. Lett. B **378** (1996) 287 [hep-ph/9602277];
D. Graudenz, hep-ph/9710244;
B. Pötter, Comput. Phys. Commun. **119** (1999) 45 [hep-ph/9806437].
- [4] A. Aktas *et al.* [H1 Collaboration], Phys. Lett. B **653** (2007) 134 [arXiv:0706.3722];
C. Adloff *et al.* [H1 Collaboration], Phys. Lett. B **542** (2002) 193 [hep-ex/0206029];
C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C **19** (2001) 429 [hep-ex/0010016];
S. Chekanov *et al.* [ZEUS Collaboration], Nucl. Phys. B **765** (2007) 1 [hep-ex/0608048];
S. Chekanov *et al.* [ZEUS Collaboration], Phys. Lett. B **649** (2007) 12 [hep-ex/0701039];
S. Chekanov *et al.* [ZEUS Collaboration], Eur. Phys. J. C **23** (2002) 13 [hep-ex/0109029].
- [5] A. Aktas *et al.* [H1 Collaboration], Eur. Phys. J. C **46** (2006) 27 [hep-ex/0508055];
S. Chekanov *et al.* [ZEUS Collaboration], Phys. Lett. B **632** (2006) 13 [hep-ex/0502029].
- [6] M. Gouzevitch [H1 Collaboration], Proceedings of the 16th International Workshop on “Deep Inelastic Scattering and QCD (DIS 08)”, London, 2008, eds. R. Devenish, M. Wing and R. Thorne, 171;
T. Kluge, *ibid.*, 172.
- [7] A.D. Martin, W.J. Stirling, R.S. Thorne and G. Watt, Phys. Lett. B **652** (2007) 292 [arXiv:0706.0459];
S. Alekhin, K. Melnikov and F. Petriello, Phys. Rev. D **74** (2006) 054033 [hep-ph/0606237];
P. Jimenez-Delgado and E. Reya, arXiv:0810.4274;
A.D. Martin, W.J. Stirling, R.S. Thorne and G. Watt, arXiv:0901.0002.
- [8] S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B **688** (2004) 101 [hep-ph/0403192];
A. Vogt, S. Moch and J. A. M. Vermaseren, Nucl. Phys. B **691** (2004) 129 [hep-ph/0404111].
- [9] E. B. Zijlstra and W. L. van Neerven, Nucl. Phys. B **383** (1992) 525; Phys. Lett. B **297** (1992) 377;
R. Hamberg, W. L. van Neerven and T. Matsuura, Nucl. Phys. B **359** (1991) 343 **644** (2002) 403 (E);
C. Anastasiou, L.J. Dixon, K. Melnikov and F. Petriello, Phys. Rev. D **69** (2004) 094008 [hep-ph/0312266];
K. Melnikov and F. Petriello, Phys. Rev. D **74** (2006) 114017 [hep-ph/0609070];
S. Alekhin and S. Moch, Phys. Lett. B **672** (2009) 166 [arXiv:0811.1412].
- [10] K. Hagiwara and D. Zeppenfeld, Nucl. Phys. B **313** (1989) 560;
F.A. Berends, W.T. Giele and H. Kuijf, Nucl. Phys. B **321** (1989) 39;
N.K. Falck, D. Graudenz and G. Kramer, Nucl. Phys. B **328** (1989) 317.

- [11] Z. Bern, L.J. Dixon, D.A. Kosower and S. Weinzierl, Nucl. Phys. B **489** (1997) 3 [hep-ph/9610370];
Z. Bern, L.J. Dixon and D.A. Kosower, Nucl. Phys. B **513** (1998) 3 [hep-ph/9708239];
E.W.N. Glover and D.J. Miller, Phys. Lett. B **396** (1997) 257 [hep-ph/9609474];
J.M. Campbell, E.W.N. Glover and D.J. Miller, Phys. Lett. B **409** (1997) 503 [hep-ph/9706297].
- [12] L.W. Garland, T. Gehrmann, E.W.N. Glover, A. Koukoutsakis and E. Remiddi, Nucl. Phys. B **627** (2002) 107 [hep-ph/0112081].
- [13] L.W. Garland, T. Gehrmann, E.W.N. Glover, A. Koukoutsakis and E. Remiddi, Nucl. Phys. B **642** (2002) 227 [hep-ph/0206067].
- [14] Z. Nagy and Z. Trocsanyi, Phys. Rev. Lett. **87** (2001) 082001 [hep-ph/0104315].
- [15] E. Mirkes and S. Willfahrt, Phys. Lett. B **414** (1997) 205 [hep-ph/9708231].
- [16] E. Mirkes and S. Willfahrt, Proceedings of the Workshop on “Physics with Polarized Protons at HERA”, eds. A. De Roeck and T. Gehrmann, DESY-Proceedings 1998-01, p. 69 [hep-ph/9711434].
- [17] Z. Kunszt and D.E. Soper, Phys. Rev. D **46** (1992) 192;
S. Frixione, Z. Kunszt and A. Signer, Nucl. Phys. B **467**, 399 (1996) [hep-ph/9512328];
S. Catani and M.H. Seymour, Nucl. Phys. B **485** (1997) 291; **510** (1997) 503(E) [hep-ph/9605323];
J. Campbell, M.A. Cullen and E.W.N. Glover, Eur. Phys. J. C **9** (1999) 245 [hep-ph/9809429];
D.A. Kosower, Phys. Rev. D **57** (1998) 5410 [hep-ph/9710213]; Phys. Rev. D **71** (2005) 045016 [hep-ph/0311272];
G. Somogyi and Z. Trocsanyi, Acta Phys. Chim. Debr. **XL** (2006) 101 [hep-ph/0609041].
- [18] A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, JHEP **0509** (2005) 056 [hep-ph/0505111].
- [19] A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, Nucl. Phys. B **691** (2004) 195 [hep-ph/0403057]; Phys. Lett. B **612** (2005) 36 [hep-ph/0501291]; **612** (2005) 49 [hep-ph/0502110].
- [20] A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover and G. Heinrich, JHEP **0711** (2007) 058 [arXiv:0710.0346]; Phys. Rev. Lett. **100** (2008) 172001 [arXiv:0802.0813].
- [21] S. Weinzierl, Phys. Rev. Lett. **101** (2008) 162001 [arXiv:0807.3241].
- [22] S. Weinzierl, arXiv:0904.1145.
- [23] A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover and G. Heinrich, Phys. Rev. Lett. **99** (2007) 132002 [arXiv:0707.1285]; JHEP **0712** (2007) 094 [arXiv:0711.4711]; arXiv:0903.4658.
- [24] S. Weinzierl, arXiv:0904.1077.
- [25] W.T. Giele, D.A. Kosower and P.Z. Skands, Phys. Rev. D **78** (2008) 014026 [arXiv:0707.3652].
- [26] S. Catani, L. Trentadue, G. Turnock and B.R. Webber, Nucl. Phys. B **407** (1993) 3.
- [27] T. Becher and M.D. Schwartz, JHEP **0807** (2008) 034 [arXiv:0803.0342].
- [28] T. Gehrmann, G. Luisoni and H. Stenzel, Phys. Lett. B **664** (2008) 265 [arXiv:0803.0695].
- [29] A. Daleo, T. Gehrmann and D. Maître, JHEP **0704** (2007) 016 [hep-ph/0612257].
- [30] T. Binoth and G. Heinrich, Nucl. Phys. B **585** (2000) 741 [hep-ph/0004013];
G. Heinrich, Nucl. Phys. Proc. Suppl. **116** (2003) 368 [hep-ph/0211144];
A. Gehrmann-De Ridder, T. Gehrmann and G. Heinrich, Nucl. Phys. B **682** (2004) 265 [hep-ph/0311276];
C. Anastasiou, K. Melnikov and F. Petriello, Phys. Rev. D **69** (2004) 076010 [hep-ph/0311311];
T. Binoth and G. Heinrich, Nucl. Phys. B **693** (2004) 134 [hep-ph/0402265].

- [31] C. Anastasiou, K. Melnikov and F. Petriello, Phys. Rev. Lett. **93** (2004) 262002 [hep-ph/0409088]; Nucl. Phys. B **724** (2005) 197 [hep-ph/0501130];
C. Anastasiou, G. Dissertori and F. Stockli, JHEP **0709** (2007) 018 [arXiv:0707.2373].
- [32] K. Melnikov and F. Petriello, Phys. Rev. Lett. **96** (2006) 231803 [hep-ph/0603182].
- [33] M. Grazzini, JHEP **0802** (2008) 043 [arXiv:0801.3232];
S. Catani, L. Cieri, G. Ferrera, D. de Florian and M. Grazzini, arXiv:0903.2120.
- [34] H. Weyl, *Gruppentheorie und Quantenmechanik* (Leipzig, 1928); B.L. van der Waerden, Göttinger Nachrichten, 100 (1929).
- [35] F. A. Berends and W. Giele, Nucl. Phys. **B294** (1987) 700.
- [36] C.G. Bollini and J.J. Giambiagi, Nuovo Cim. **12B** (1972) 20;
G.M. Cicuta and E. Montaldi, Nuovo Cim. Lett. **4** (1972) 329.
- [37] G. 't Hooft and M. Veltman, Nucl. Phys. B **44** (1972) 189.
- [38] F.V. Tkachov, Phys. Lett. B **100** (1981) 65;
K.G. Chetyrkin and F.V. Tkachov, Nucl. Phys. B **192** (1981) 159.
- [39] T. Gehrmann and E. Remiddi, Nucl. Phys. B **580** (2000) 485 [hep-ph/9912329].
- [40] S. Laporta, Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033].
- [41] *MAPLE V Release 7*, Copyright 2001 by Waterloo Maple Inc.
- [42] J.A.M. Vermaseren, *Symbolic Manipulation with FORM*, Version 2, CAN, Amsterdam, 1991 and Version 3, math-ph/0010025.
- [43] T. Gehrmann and E. Remiddi, Nucl. Phys. B **601** (2001) 248 [hep-ph/0008287]; **601** (2001) 287 [hep-ph/0101124].
- [44] S. Moch, P. Uwer and S. Weinzierl, Phys. Rev. D **66** (2002) 114001 [hep-ph/0207043].
- [45] E. Remiddi and J.A.M. Vermaseren, Int. J. Mod. Phys. A **15** (2000) 725 [hep-ph/9905237].
- [46] T. Gehrmann and E. Remiddi, Comput. Phys. Commun. **141** (2001) 296 [hep-ph/0107173].
- [47] D. Maître, Comput. Phys. Commun. **174** (2006) 222 [hep-ph/0507152]; hep-ph/0703052.
- [48] T. Gehrmann and E. Remiddi, Comput. Phys. Commun. **144** (2002) 200 [hep-ph/0111255].
- [49] T. Gehrmann and E. Remiddi, Nucl. Phys. B **640** (2002) 379 [hep-ph/0207020].
- [50] C. Anastasiou and A. Daleo, JHEP **0610** (2006) 031 [hep-ph/0511176].
- [51] S. Catani, Phys. Lett. B **427** (1998) 161 [hep-ph/9802439];
G. Sterman and M. E. Tejeda-Yeomans, Phys. Lett. B **552** (2003) 48 [hep-ph/0210130];
T. Becher and M. Neubert, arXiv:0901.0722; arXiv:0903.1126.
- [52] R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B **178** (1981) 421.
- [53] W.T. Giele and E.W.N. Glover, Phys. Rev. D **46** (1992) 1980.